

THE SAGE ENGINE

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1. SAGE

Sage is a mathematical software system providing an open source alternative to Magma, Maple, Mathematica, and Matlab. It is available from <http://www.sagemath.org/> as a free download. The program provides all of the standard features expected from such a system: arbitrary precision arithmetic, symbolic integration and differentiation, two-dimensional plotting of functions, matrix algebra, and much more.

Download sage from this web site. Move the resulting “sage” directory to /Applications. Open the sage directory and double click the “sage” icon to initialize sage.

SageTeX is a piece of the Sage download. It is basically a LaTeX style file, which allows users to embed and process Sage code from within TeX files. The last three pages of this document show this style file in action. The source code on page three is followed by LaTeX output on pages four and five. In the source file, the initial line

```
% !TEX TS-program = sage
```

tells TeXShop to process the file using the sage engine; this engine first calls pdfflatex, then calls sage, and finally calls pdfflatex again. The remaining lines in the preamble are standard LaTeX commands, except the required line

```
\usepackage{sagetex}
```

In the remaining source, sage commands are entered within lines of the form

```
\sage{...}
```

These line cause sage to process commands and output LaTeX source fragments, which become part of the LaTeX document.

Notice in particular that sage can plot standard functions. Sage can also compute integrals symbolically; for example, look carefully at the command which processes $\int \frac{x^2+x+1}{(x-1)^3(x^2+x+2)}$. This command contains standard LaTeX code to display the integral, but then Sage integrates and returns a typeset copy of the result.

2. SETTING UP THE ENGINE

This folder contains an engine file named “sage.engine”. Move this file to the active portion of `~/Library/TeXShop/Engines`. This engine file assumes that Sage was installed in `/Applications`. If Sage was installed in a different location, edit `sage.engine` appropriately. Dan Drake, who is responsible for SageTeX, wrote this engine.

One final step is required to use SageTeX. The “sage” directory contains a style file named “sagetex.sty” and a number of support files. These files must be copied to your TeX distribution. The files depend on other features of sage, so whenever you upgrade sage, you also need to upgrade the copies in your TeX distribution.

Read the sage web page <http://sagemath.org/doc/installation/sagetex.html> for instructions on copying the files. It is easiest to install them in `~/Library/texmf`. If you installed sage in `/Applications`, the web page instructions are easily summarized: open Terminal and issue the command

```
cp -R /Applications/sage/local/share/texmf/tex ~/Library/texmf
```

There is one final problem. The release version of TeX Live 2009 contained `sagetex.sty`. This file is updated whenever sage is updated, so the copy in TeX Live easily gets out of sync with sage. Therefore the authors of sage requested that TeX Live remove their copy of `sagetex.sty`. If you updated TeX Live 2009 with TeX Live Utility or `tlmgr`, it should be gone. But just to make sure, type the following commands in Terminal:

```
cd /usr/local/texlive/2009/texmf-dist/tex/latex/sagetex
ls
```

If this location contains `sagetex.sty`, type

```
sudo mv sagetex.sty sagetex-old.sty
```

Experts will remark that it shouldn’t make any difference because `~/Library/texmf` takes precedence. True enough for TeX, but experiments show that the old copy `texmf-dist` breaks sage. We don’t quite know why that is.

3. FINAL REMARKS

Although spaces are permitted in LaTeX source file names, Sage does not like spaces in file names. So LaTeX source file names which call SageTeX should not contain spaces. This problem is fixed in beta releases of SageTeX not yet in the sage distribution.

A Sage tutorial is available at the Sage page <http://www.sagemath.org/help.html>. It is definitely recommended. Extensive additional documentation is available at the same web page.

```
% !TEX TS-program = sage
```

```
% The following lines are standard LaTeX preamble statements.
```

```
\documentclass[11pt, onside]{amsart}
\usepackage{geometry}
\geometry{letterpaper}
\usepackage[parfill]{parskip}
\usepackage{graphicx}
\usepackage{amssymb}
\usepackage{epstopdf}
\title{Brief Article}
\author{The Author}
```

```
% Only one command is required to use Sage within the LaTeX source:
```

```
\usepackage{sagetex}
```

```
\begin{document}
```

```
\maketitle
```

```
\section{Introduction}
```

This is an example of using Sage within a \TeX document. We can compute extended values like

```
$$32^{31} = \sage{32^31}$$
```

We can plot functions like $x \sin x$:

```
\sageplot[width=5in]{plot(x * sin( 30 * x), -1, 1)}
```

We can integrate:

```
$$\int \frac{x^2 + x + 1}{(x - 1)^3 (x^2 + x + 2)} dx = \sage{integrate( (x^2 + x + 1) / ((x - 1)^3 * (x^2 + x + 2)) )}$$
```

```
\newpage
```

We can perform matrix calculations:

```
$$\sage{matrix([[1, 2, 3], [4, 5, 6], [7, 8, 9]]^3)}$$
```

```
$$AB = \sage{Matrix([[1, 2], [3, 4]])} \sage{Matrix([[5, 6], [6, 8]])} = \sage{Matrix([[1, 2], [3, 4]]) * Matrix([[5, 6], [6, 8]])}$$
```

Plots are fun; here is a second one showing $x \ln x$. The "width" command in the source is sent to the include graphics command in LaTeX rather than to Sage.

```
\sageplot[width=5in]{plot(x * ln(x), 0, 2)}
```

Sage understands mathematical constants and writes them symbolically unless it is told to produce a numerical approximation. The term π below is not in the LaTeX source; instead it is the result of a Sage calculation, as is the numerical value on the other side of the equal sign.

The product of e and π is $\sage{\pi * e} = \sage{N(\pi * e)}$.

```
\end{document}
```

BRIEF ARTICLE

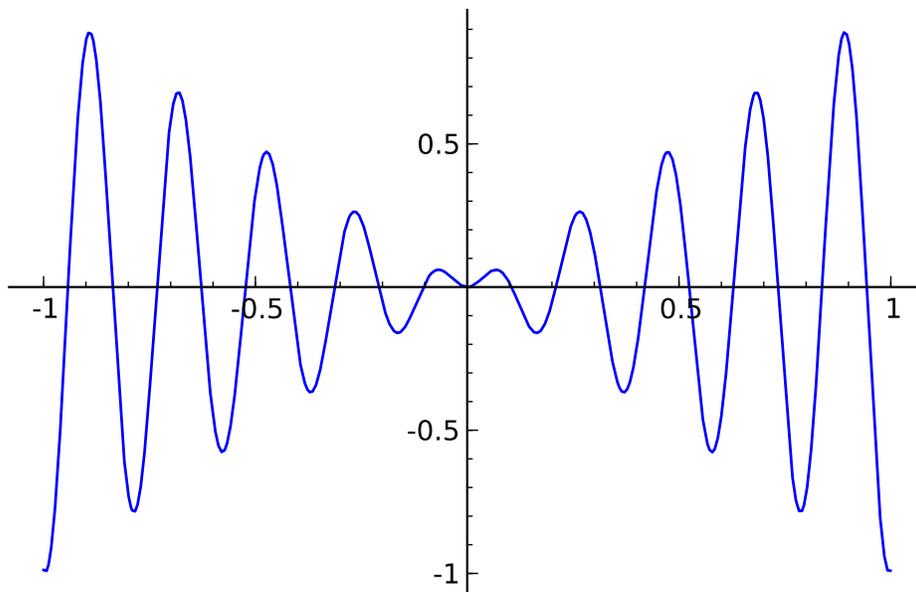
THE AUTHOR

1. INTRODUCTION

This is an example of using Sage within a \TeX document. We can compute extended values like

$$32^{31} = 45671926166590716193865151022383844364247891968$$

We can plot functions like $x \sin x$:



We can integrate:

$$\int \frac{x^2 + x + 1}{(x-1)^3(x^2 + x + 2)} dx$$
$$= -\frac{9}{448} \sqrt{7} \arctan\left(\frac{1}{7}(2x+1)\sqrt{7}\right) - \frac{3}{16} \frac{(x+1)}{(x^2 - 2x + 1)} - \frac{5}{64} \log(x-1) + \frac{5}{128} \log(x^2 + x + 2)$$

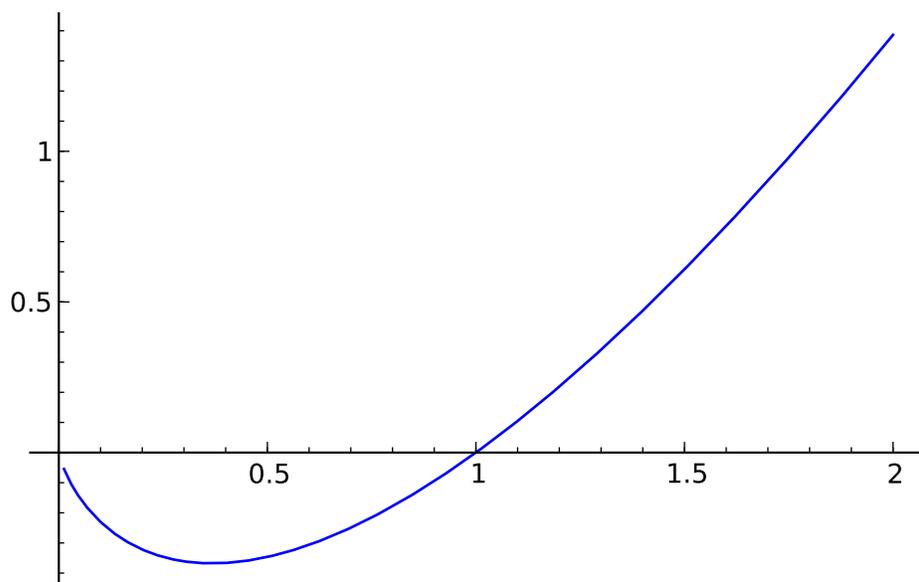
1

We can perform matrix calculations:

$$\begin{pmatrix} 468 & 576 & 684 \\ 1062 & 1305 & 1548 \\ 1656 & 2034 & 2412 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 6 & 8 \end{pmatrix} = \begin{pmatrix} 17 & 22 \\ 39 & 50 \end{pmatrix}$$

Plots are fun; here is a second one showing $x \ln x$. The “width” command in the source is sent to the include graphics command in LaTeX rather than to Sage.



Sage understands mathematical constants and writes them symbolically unless it is told to produce a numerical approximation. The term $e\pi$ below is not in the LaTeX source; instead it is the result of a Sage calculation, as is the numerical value on the other side of the equal sign.

The product of e and π is $\pi e = 8.53973422267357$.